Coexistence in May-Leonard Model with Random Rates

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Introduction

The rock–paper–scissors (RPS) system without conservation law or, equivalently, May-Leonard model, has been widely studied in order to understand biodiversity in ecology and biology. As a non-trivial model for cyclic competition, RPS constitutes paradigmatic model system to mathematically describe the co-evolutionary dynamics of three coexisting species in cyclic competition, such as, e.g., realized in nature for three types of Californian lizards, and the coexistence of three strains of E. coli bacteria in microbial experiments. Among the various RPS variants, a particular model introduced by May and Leonard has recently received much attention, leading to novel results that have important implications for the formation and propagation of spatial patterns in ecological systems [1,2,3]. Our research is to study the effect of spatial disorder on the co-existence of the regular May-Leonard systems.

May-Leonard Model

Model definition. The model is defined through the following set of binary predation, first-order offspring production reactions between the three particle species, and exchange and hopping processes:

\[ A + B \rightarrow \emptyset + A; \text{with rate } \sigma \]
\[ B + C \rightarrow \emptyset + B; \text{with rate } \sigma \]
\[ C + A \rightarrow \emptyset + C; \text{with rate } \sigma \]
\[ X + \emptyset \rightarrow X + X; \text{with rate } \mu \]
\[ X + Y \rightarrow Y + X; \text{with exchange rate } \epsilon \]
\[ X + \emptyset \rightarrow X + X; \text{with hopping rate } D \]

Where \( X \) and \( Y \) refer to any one of the three species \( A, B, \) and \( C \).

Quantities of interest: snapshots of spatial particle distribution, time–dependent population densities \( a(t) \), the corresponding temporal Fourier transform, and the equal-time two-point occupation number correlation functions \( C_{\mu\nu} \) and \( C_{\epsilon\epsilon} \).

1. Snapshots of the spatial particle distribution at \( t=1000 \) MCS for a system with \( N = 256 \times 256 \) sites with equal initial densities \( a(0) = 1/3 \).

2. Quantitative observables at \( t=1000 \) MCS for a system with \( N = 256 \times 256 \) sites and with random rates.

3. Mean extinction times and their distribution.

Conclusion: firstly, quenched spatial disorder in either the reaction or the mobility rates does not significantly affect the temporal evolution, Fourier signals, or spatial correlation functions in stochastic spatial May–Leonard models. Secondly, particle pair exchange processes promote the formation of sharp spiral patterns. Moreover, there exits a remarkable gradual transformation in the dependence of the mean extinction time on system size, and the shape of the associated extinction time distribution, when the effective mobility rate crosses the critical threshold.

Bibliography